Formulating LUTI calibration as an optimisation problem: example of Tranus shadow-price estimation

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1 Introduction

Integrated land use and transport modelling has attracted the attention of researchers since 1960 [4]. Over the years, a large number of models have come into existence. It is well known that integration of land use and transport models creates a complex nonlinear system, which evolves in different scales. Analysing these complex systems is typically a hard problem, especially in the presence of uncertainty, whose effects may be difficult to assess. In such cases, calibration plays a central role, as it helps us determine optimal parameters. Calibrating this type of models is a process that requires many steps. At first, the data needed is not always readily available, but even if so, finding the set of parameters that replicates better the data is not simple. We propose an optimisation framework for the calibration of Tranus, particularly for obtaining a good estimation of the shadow-price variables.

2 Methodology

Tranus Description. Tranus [3] provides a generic framework to model land use and transportation in an integrated manner, both in urban and regional levels. The region of interest is divided into *economic sectors* and *spatial zones*. Then Tranus combines two modules: the *land use and activity* module which simulates a spatial economic system by assessing the activity locations and economic sector interactions (LCAL); and a *transportation* module, which estimates the use of the transport network and the associated disutility (LOC).

The land use and activity module estimates the productions and the consumptions for zones at a given period, and the demands of flows that this activity generates. These demands of flows are then fed to the transportation module. In this way the movements of people or freight are explained as the results of the economic and spatial interaction between activities, the transport system and the real estate market. Then from the transportation network, once the corresponding trips are generated, travel flows are allocated to the network according to travel demand. In turn, the accessibility that results from the transport system influences the location and interaction between activities through transport dis-utilities, also affecting land rent.

The two modules in the system use discrete choice logit models [2], linked together in a consistent way. This includes activity-location, land-choice, and multi-modal path choice and assignment. The modules are then run iteratively, such that production and consumption demands for each zone are met and equilibrium is achieved. A detail description of the equations of Tranus land use and transportation modules can be found in [3].

The problem. In Tranus, the calibration of the land-use and acitivity module is done in a program called LCAL (Land-use CALibration). The objective of the calibration, is to have a model that can replicate the data of the base year. For achieving this, the model has a set of parameters that can be adjusted (elasticities, attractors, discrete choice models constants, etc.). The job of the modeler, is to adjust the input parameters, to make the model fit the base year production data X_0 as close as possible. To be sure that the model can replicate the base year's production, a correction term is added inside LCAL. This variable is added to the utility function and acts as a correcting price, called the shadow-price. One shadow-price is added for each production. What LCAL does in the actual implementation of Tranus, is for any given set of parameters, it will *optimise* the shadow-prices to replicate the base year X_0 for a fixed set of input parameters. This is done via an iterative method that tries to find a local optimum just exploiting the economic notion of the price and shadow-price variable, there is an explicit cost function. This can lead to local optima, or not even find a solution.

Instead, we propose a framework, with an actual cost function to optimise and a clear and explicit description of the problem.

The problem to solve in LCAL, is to adjust the shadow-prices $h \in \mathbb{R}^n \times \mathbb{R}^j$ (n sectors, j zones) to replicate the base year production. The problem can be written as an optimisation problem:

$$\min_{h} \quad f(h) = \|X(h) - X_0\|^2 \tag{1}$$

The function $X(h)$ doesn't have a closed form, because it is the result of a double fixed point problem:

$$X(h) \ = \Lambda_1(h,p) + \Delta_1(h,p)X \tag{2}$$

$$p(h) \ = \Lambda_2(h,p) + \Delta_2(h,p)\ p \tag{3}$$

Where equation (3) represents the computation of the intermediate variable p (price), and imposes that the prices are in equilibrium. The problem here is that for computing $X(h)$, we need to solve (2)-(3) iteratively until convergence. Instead of imposing that the prices are in equilibrium, we can add it to the objective function, hence we can rewrite the problem as follows:

$$\min_{h,p} \quad f(h,p) = \|\hat{X}(h,p) - X_0\|^2 + \|\hat{p}(h) - p\|^2 \tag{4}$$

where $\hat{X}$ is the solution of the linear system (2) and $\hat{p}$ is the solution of (3), X_0 is the observed induced production.
This appears to be a good approach to attain reasonable shadow prices, but the construction of the Δ_1 and Δ_2 matrices and the inversion of linear systems (2)-(3) is expensive. That's why we propose another idea, based on the concept of imposing X_0 as fixed point for the linear system.
Instead of solving the linear systems, we evaluate them in X_0. We exploit the idea that that if we could find a pair (h^*, p^*) so that $f(h^*, p^*) = 0$, then $\hat{X}(h^*, p^*) = X_0$ and $\hat{p}(h^*) = p^*$. We define the functions:

$$\tilde{X}(h,p) = \Lambda_1(h,p) + \Delta_1(h,p)X_0 \tag{5}$$

$$\tilde{p}(h,p) = \Lambda_2(h,p) + \Delta_2(h,p)\ p \tag{6}$$

Then, our optimisation problem is as follows,

$$\min_{h,p} \quad f(h,p) = \|\tilde{X}(h,p) - X_0\|^2 + \|\tilde{p}(h,p) - p\|^2 \tag{7}$$

Test data. It general terms, is difficult to evaluate a calibration. This difficulty is realated to the fact that we dont know the optimal set of parameters. To be able to test our optimisation scheme, we constructed a set of input data $(X_0, parameters)$ that has a perfect calibration (up to round-off error). In other words, for an specified shadow-price h, the calculated $X(h) = X_0$. This is a practical for two reasons. First, we are sure to know the optima, so we can make plots of the function around the optimal value. Secondly, asessing the robustness of the optimisation algorithm, testing different starting points and checking if the convergence to the optimal value is attained. To generate the test files, we start with a initial h (ideally a vector of zeros), and we compute iteratively the production (2) and price (3) to attain convergence between successive iterations, then we replace the input data X_0 by the estimated production $X(h)$, thus generating a *perfect* data set.

3 Results

In terms of optimisation, the functions Δ and Λ are differentiable, even class C^∞, so we develop an optimisation framework using the optimisation algorithm presented in [1]. The Jacobian of the function is computed analytically.
Example: We tried a small example, based in the *Example_C* from the Tranus website (`http://www.tranus.com/tranus-english/download-install`). We generated the *perfect* files, with shadow-prices $h_i^n = 0$ for each sector n and zone i. This example, has 3 zones and 5 sectors. We did some cuts near the optimal value, and plotted the function $f(h,p)$. In the Figure 1, the graph set is generated evaluating the function f for shadow-prices h of the first sector and the three different zones. As expected, the cost function is zero at $h = 0$, and increases its value when we get far away from the optima. The function appears to be locally convex near the optimal value (at least for these 3 parameters)

What is really interesting, is to have a plot of the function f near the optimum, for a fixed sector and zone, and see how the function behaves with h and p. If we consider sector 1 and zone 2, we can

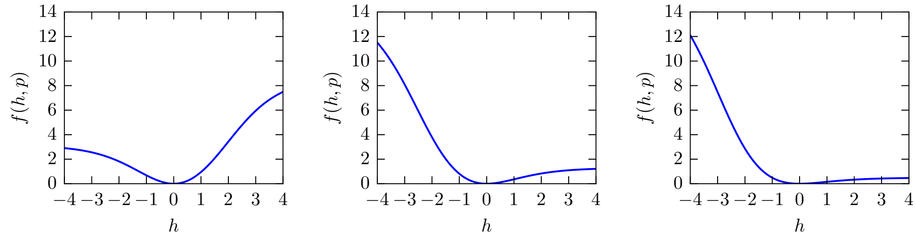

Figure 1: Graph of slices function f along h_i^1, for each zone $i \in [1, 2, 3]$

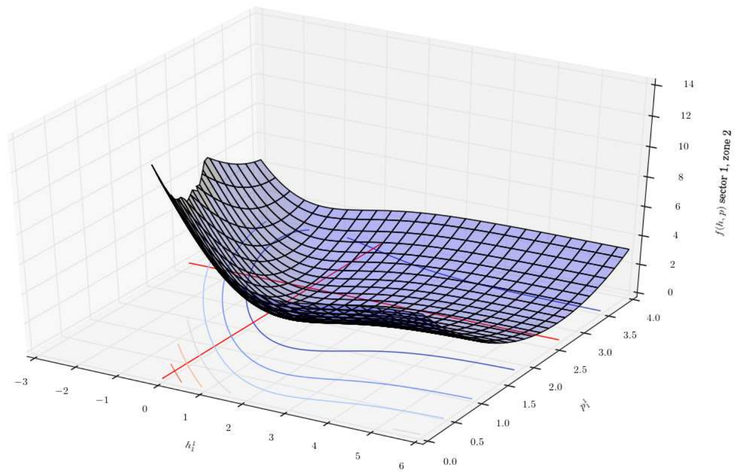

Figure 2: Plot of function f for a given pair (h_2^1, p_2^1) near the optimal value (depicted in red)

plot f near the optimal value $h_1^1 = 0, p_1^1 = 2.676$ as shown in Figure 2. Here we can observe that as the shadow-price gets larger the value of f increases up to a plateau state ($X_1^1(h) \to 0$ and then $f(h, p)$ becomes constant). In the case of the price p, if we move away from the optimal value $p = 2.676$, the cost increases quadratically.

We tested the robustness of the optimisation scheme with 1000 random initial values, always converging to the same optimum. The prices are in the interval $[0, 4]$, so considering the shadow prices in $[-10, 10]$ is highly representative.

4 Conclusions

We have a developed a optimisation methodology that give us a partial calibration of Tranus and a clear representation near the optimal value. Secondly, we have proposed a technical contribution to the way equilibrium equations (2)-(3) are handled in Tranus, permiting us to decouple the double point fix problem. Finally, the procedure of generating syntethic data files for testing the calibration methodology is a practical way to test and check the performance of a general calibration scheme.

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